Schiff Theorem and the EDMs of H-Like Atoms

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Why?

- A permanent EDM violates both P and T invariances.
- By CPT invariance, $\not{T} \equiv \not{C}\not{P}$.
- A neutral system is easier for measurement.

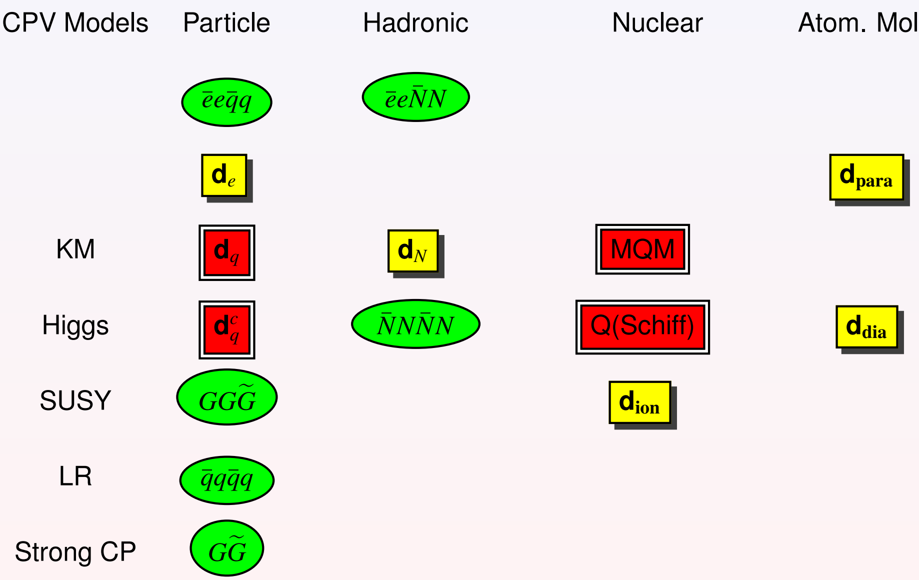
What they tell?

An atom is composed by electrons and nucleons, so its EDM receives contributions from

1. Electron EDM: d_e
2. Nucleon EDMs: $d_N = d_{n,\,p}$
3. Semi-Leptonic $\not{P}\not{T}$ interactions: $C^{eN}_{\rm PS,S}$, $C^{eN}_{\rm S,PS}$, $C^{eN}_{\rm PV,V}$, $C^{eN}_{\rm V,PV}$, and $C^{eN}_{\rm T,PT}$
4. Hadronic $\not{P}\not{T}$ interactions: (i) $C^{NN}_{\rm S,PS}$ and $C^{NN}_{\rm V,PV}$ or (ii) $\bar{g}_{\pi,\,\eta,\,\rho,\,\omega\ldots}$

Ultimate goal

To express $d_A = d_A(d_e,\, d_N,\, \bar{g}_M,\, C^{eN}\ldots) = d_A(d_e, \bar{c}_{eq}\, d_q,\, d^c_q,\, w,\, \bar{c}_{4q},\, \theta\ldots)$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Strong CP
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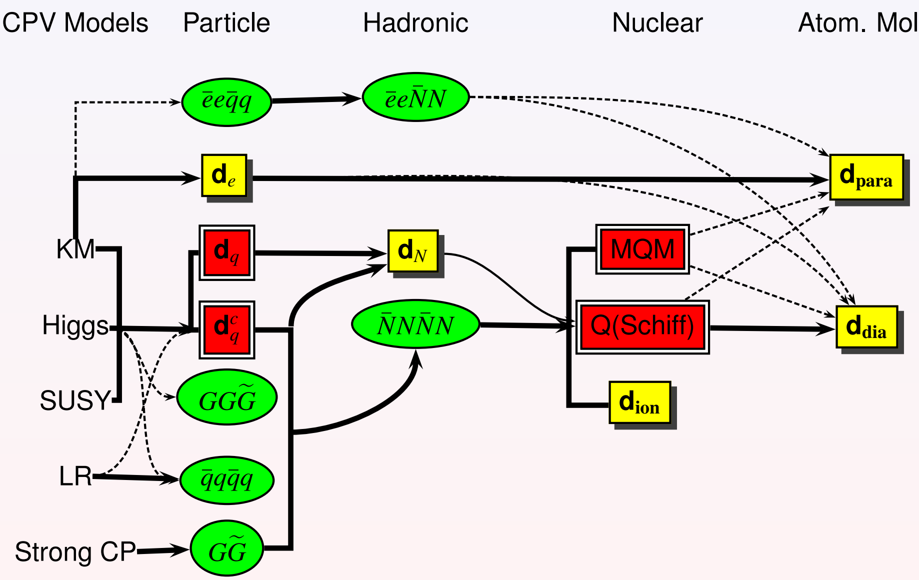
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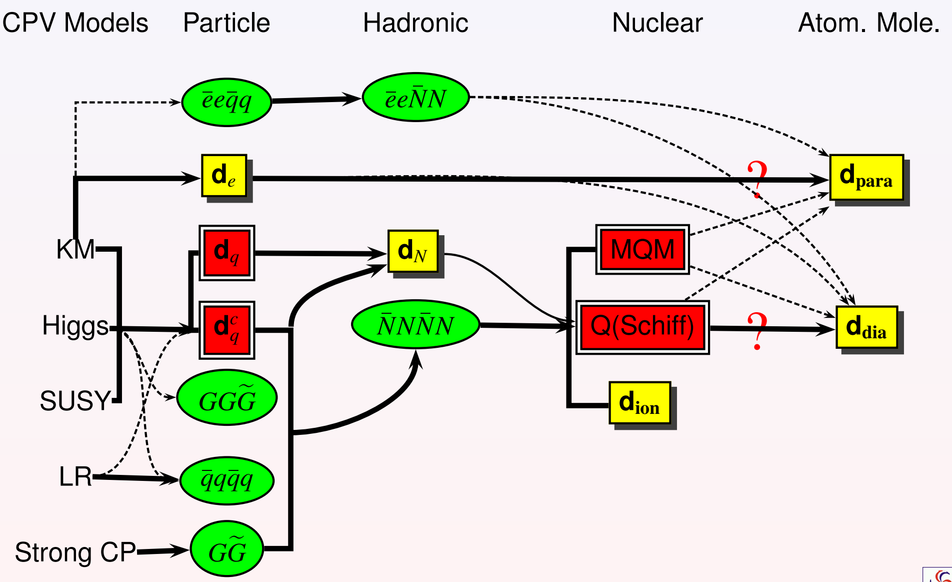
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Theorem

For a NR system made up of point, charged particles which interact electrostatically with each other and with an arbitrary external field, the shielding is complete. (Schiff, 1963)

- Classical picture: The re-arrangement of constituent charged particles in order to keep the whole system stationary
- Quantum-Mechanical description: Schiff (1963), Sandars (1968), Feinberg (1977), Sushkov, Flambaum, and Khriplovich (1984), Engel, Friar, and Hayes (2000), Flambaum and Ginges (2002) ...

What this implies?

- The measurability of atomic EDMs is severely constrained.
- One has to look for the loopholes (Sc63, Sa68) in
  - relativistic effects (electron)
  - finite-size effects (nucleus)
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Assuming one-electron only (a generalization to multi-electron is straightforward):

$$\langle \boldsymbol{d}_A \rangle = \underbrace{\langle \beta \boldsymbol{d}_e \rangle + \langle \boldsymbol{d}_{\mathcal{N}} \rangle}_{\text{intrinsic}} + \underbrace{\sum_n \frac{e}{\Delta E_n} \langle \text{g.s.}| H^{(\text{int})}_{\not P \not T} |n\rangle \langle n| \boldsymbol{x} |\text{g.s.}\rangle + \text{c.c.}}_{\text{polarized}}$$

Shielding of d_e

$$H^{(d_e)}_{\not P \not T} = -\beta \boldsymbol{d}_e \cdot \boldsymbol{E}_{\mathcal{N}} = \underbrace{[-\beta \boldsymbol{d}_e \cdot \nabla \,,\, H_0]/e}_{(1)} + \underbrace{\Delta H^{(d_e)}_{\not P \not T}}_{(2)}$$

1. Leads to $\langle [-\beta \boldsymbol{d}_e \cdot \nabla \,,\, \boldsymbol{x}] \rangle = -\langle \beta \boldsymbol{d}_e \rangle$: the shielding!
2. Prop. to γ_5, vanishing in the NR limit (no small component): equiv. to the $(\beta - 1)$ formalism (Sa68)

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Shielding of $d_{\mathcal{N}}$

$$H_{\not P \not T}^{(d_{\mathcal{N}})} = -\boldsymbol{d}_{\mathcal{N}} \cdot \boldsymbol{E}_e = \underbrace{[-\boldsymbol{d}_{\mathcal{N}} \cdot \boldsymbol{\nabla}\, ,\, H_0]/(Z\,e)}_{(1)} + \underbrace{\Delta H_{\not P \not T}^{(d_{\mathcal{N}})}}_{(2)}$$

1. Leads to $\langle[-\boldsymbol{d}_{\mathcal{N}} \cdot \boldsymbol{\nabla}\, ,\, \boldsymbol{x}]\rangle = -\langle \boldsymbol{d}_{\mathcal{N}}\rangle$: the shielding!
2. Contains composite operators: $\boldsymbol{d}_{\mathcal{N}} \otimes C_{2,4\ldots}$, $\boldsymbol{d}_{\mathcal{N}} \otimes M_{1,3\ldots}$, $\boldsymbol{d}_{\mathcal{N}} \otimes \mathcal{C}_{0,2,4\ldots}(x)$, and $\boldsymbol{d}_{\mathcal{N}} \otimes \mathcal{M}_{1,3\ldots}(x)$.

note: By definition, $d_{\mathcal{N}} \equiv C_1$, so $H_{\not P \not T}$ due to $\mathcal{C}_1(x)$ should be considered as extra.

Caution

1. Above derivation is purely quantum-mechanical, that is, all physical observables are OPERATORS.
   The atomic/nuclear matrix elements are only taken at the final stage.
2. $\boldsymbol{d}_e = d_e\,\boldsymbol{\sigma}$ and $\boldsymbol{d}_{\mathcal{N}} = \sum_{i=1}^{A}(d_s + d_v\,\tau_i^z)/2\,\boldsymbol{\sigma}_i \quad + \quad \underbrace{e\,\boldsymbol{r}_i}_{\text{need } \not P \not T\ NN \text{ force}}$

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Definition

A residual nuclear charge moment with $J = 1$ (EDM-like) after the shielding takes effect (SFK84), which contains operators $\mathcal{C}_1(x)$ and some terms in $[\boldsymbol{d}_{\mathcal{N}} \otimes \mathcal{C}_{0,2}(x)]_1$.

- Finite-size effect is manifest
- Needs atomic w.f. inside the nucleus (FG02)

$$\boldsymbol{S} = \sum_{k=1}^{\text{odd}} \frac{b_k}{b_1} \frac{1}{(k+1)\,(k+4)} \sum_{i=1}^{A} \left(y_i^{k+1} \boldsymbol{y}_i - \frac{(k+4)}{3} \frac{1}{Z} \left[y_i^{k+1} (1 - \frac{4\sqrt{\pi}}{5} Y_2(\hat{y}_i)) \otimes \boldsymbol{d}_{\mathcal{N}} \right]_1 \right)$$

$$\approx \frac{1}{10} \sum_{i=1}^{A} \left(y_i^2 \boldsymbol{y}_i - \frac{5}{3\,Z} [y_i^2 \otimes \boldsymbol{d}_{\mathcal{N}}]_1 + \frac{4\sqrt{\pi}}{3\,Z} [y_i^2 Y_2(\hat{y}_i) \otimes \boldsymbol{d}_{\mathcal{N}}]_1 \right)$$

Difference from existing literature

1. How good is $\langle A \otimes B \rangle = \langle A \rangle \otimes \langle B \rangle$?
2. Quadrupole deformation is taken into account for $I > 1/2$

note: For deuteron: 1 : -1.67 : -1.33 (new) vs. 1 : -0.59 : -0.071 (old)

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- For a paramagnetic system, all $\not{P}\not{T}$ ingredients including
  - d_e: partially shielded
  - C^{eN}: not affected by shielding
  - $\not{P}\not{T}$ nuclear charge moments : S (partially shielded) and C_3...
  - $\not{P}\not{T}$ nuclear magnetic moments: $[d_{\mathcal{N}} \otimes \mu_{\mathcal{N}}]$ (due to re-arrangement) and M_2...

  all come into play. Which one dominates?
- H-like atoms (only H is neutral) are the simplest paramagnetic systems, where calculations can be simply performed and show some systematics
  - solve Sternheimer equation: $\sum_{J'} |\widetilde{J}', J\rangle = \sum_n \frac{-1}{\Delta E_n} |n\rangle \langle n| z |J, J\rangle$

$$\langle d_A \rangle = \sum_{J'} \langle I, I| \otimes \langle J, J| H_{\not{P}\not{T}} |\widetilde{J}', J\rangle \otimes |I, I\rangle + \text{c.c.}$$

  - atomic ground state is $1s_{1/2}$, so $\widetilde{J}' = 1/2$ or $3/2$
  - assuming Pauli approximation, analytical results are possible

note: The solution $|\widetilde{J}', J\rangle$ only depends on atomic physics, has nothing to do with $H_{\not{P}\not{T}}$.

The growth rate as Z or A increases

$$d_A(d_e : C^{eN}_{\mathrm{PS,S}} : S : S^{\mathrm{mag}}) = \underbrace{Z}_{(1)} \times (\underbrace{Z}_{(2)} : \underbrace{A}_{(3)} : \underbrace{S}_{(4)} : \underbrace{S^{\mathrm{mag}}}_{(5)})$$

1. from the atomic structure calculation
2. the nuclear charge
3. the coherent contributions from nucleons (isoscalar)
4. $r^2\boldsymbol{r}$ in $\boldsymbol{S}$ roughly scales as $A^{2/3}$
5. M_2 in $\boldsymbol{S}^{\mathrm{mag}}$ roughly scales as $A^{2/3}$

Why not Z^3 for d_e?

- Because we consider a $1s_{1/2}$ electron, not a valence electron whose energy gap from $ns_{1/2} \to n'p$ roughly decreases as Z
- Even in the latter case, the ratio $Z : A : S : S^{\mathrm{mag}}$ is roughly unchanged
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Initial values for Z=1 with a crude estimate

- $\bar{d}_n = -\bar{d}_p \sim 0.01\,\bar{G}_\pi^{(0)}$, $d_{\mathcal{D}} \sim 0.015\,\bar{G}_\pi^{(1)}$, $\bar{G}_\pi^{(2)} \sim 0$
- $C_{\mathrm{PS,S}}^{(p,n)} \sim \frac{g_{\pi ee}}{g_{\pi NN}}\,\frac{1}{m_\pi^2}\,\frac{\sqrt{2}}{G_F}\,\bar{G}_\pi^{(p,n)} \sim -0.164\,(\pm\bar{G}_\pi^{(0)} + \bar{G}_\pi^{(1)})$

$$d_{\mathrm{H}} \sim -1.07\times 10^{-4}\,d_e - 1.58\times 10^{-11}\,(d_n + 2/3\,d_{\mathcal{D}}) + 0 - 6.72\times 10^{-8}\,d_n$$

$$d_{\mathrm{D}} \sim -1.07\times 10^{-4}\,d_e - 2.11\times 10^{-11}\,d_{\mathcal{D}} - 3.36\times 10^{-9}\,d_{\mathcal{D}} - 5.78\times 10^{-8}\,(2.90\,d_n - 0.77\,d_{\mathcal{D}})$$

Some observations

- S or S^{mag} is 2-3 orders of magnitude greater than C^{eN}, decreases as $A^{2/3}/A = A^{-1/3}$
- S^{mag} is 1-2 orders of magnitude greater than S, roughly keeps the same as long as M_2 exists
- If $d_{n,\mathcal{D}}/d_e \sim 10^3$–$10^4$ (SM gives 10^4–10^6), hadronic contributions are as large as d_e and only decrease as $A^{2/3}/Z$

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- The Schiff theorem is derived at the operator level. The Schiff operator and its matrix element, the Schiff moment, we got is different from existing literature. For a deuteron, the difference is huge, and check on nuclei of great interests like Hg, Xe, Ra ...etc. should be carried out.
- For paramagnetic atoms, semi-leptonic and hadronic contributions should be considered in order to establish how effective these atoms can be used to constrain the electron EDM. So, the Schiff moment of Tl, Schiff and MQM of Cs, ...etc. could potentially be important.